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Technical Note

Mixed convection boundary layer flow adjacent to a vertical surface embedded in a stable stratified medium

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Abstract

The steady mixed convection boundary layer flow through a stable stratified medium adjacent to a vertical surface is investigated. The velocity outside the boundary layer and the surface temperature are assumed to vary linearly from the leading edge of the surface. The transformed ordinary differential equations are solved numerically by the Keller-box method. It is found that dual solutions exist, and the thermal stratification delays the boundary layer separation.

Keywords: Boundary layer; Dual solutions; Mixed convection; Stratified medium

1. Introduction

The flow due to a heated surface immersed in a stable stratified viscous fluid has been investigated experimentally and analytically in several studies such as Yang et al. [1], Jaluria and Gebhart [2] and Chen and Eichhorn [3]. Jaluria and Gebhart [2] studied the stability of the flow adjacent to a vertical plate dissipating a uniform heat flux into a stratified medium, and found that similarity solution exists, in which the ambient stratification varies like $x^{1/5}$, where x is downstream coordinate. Kulkarni et al. [4] studied the problem of natural convection flow over an isothermal vertical wall immersed in a thermally stratified medium and reported the similarity solutions. The same problem was then investigated experimentally by Tanny and Cohen [5], who found that the local heat transfer is in good agreement with the theoretical predictions done by Kulkarni et al. [4].

This note presents the solution of the steady mixed convection boundary layer flow through a stable stratified medium adjacent to a semi-infinite vertical surface. The external velocity and the surface temperature are assumed to vary linearly with the distance measured from the leading edge of the surface.

2. Problem formulation

Consider the steady mixed convection boundary layer flow over a heated vertical flat plate of temperature $T_w(x)$, which is embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$, where $T_w(x) > T_\infty(x)$. It is assumed that $T_w(x) = T_0 + bx$, $T_\infty(x) = T_0 + cx$, and the velocity outside the boundary layer is of the form $U(x) = ax$, where a, b and c are constants (with $a > 0$, $b > 0$ and $c \geqslant 0$) and T_0 is the ambient temperature at the leading edge of the plate. This form of the ambient temperature has also been considered by Chen and Lin [6]. Under these assumptions, the boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\mathrm{d}U}{\mathrm{d}x} + \nu\frac{\partial^2 u}{\partial y^2} \pm g\beta(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

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Nomenclature

a, b, c	constants
f	dimensionless stream function
g	acceleration due to gravity
Gr_x	local Grashof number
Pr	Prandtl number
Re_x	local Reynolds number
S	stratification parameter
T	fluid temperature
T_0	ambient temperature at the leading edge
$T_w(x)$	plate temperature
$T_\infty(x)$	ambient temperature
u, v	velocity components along the x- and y-directions, respectively
$U(x)$	velocity outside the boundary layer
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
η	similarity variable
θ	dimensionless temperature
λ	buoyancy or mixed convection parameter
ν	kinematic viscosity
ψ	stream function

Subscripts

w	condition at the wall
∞	condition away from the wall

Superscript

$'$	differentiation with respect to η

subject to the boundary conditions

$$u = 0, \quad v = 0, \quad T = T_w(x) \quad \text{at} \quad y = 0, \\ u \to U(x), \quad T \to T_\infty(x) \quad \text{as} \quad y \to \infty. \tag{4}$$

The last term in Eq. (2) represents the influence of the thermal buoyancy force on the flow field with "+" and "−" signs correspond to the assisting and opposing flows, respectively.

We introduce now the following similarity variables:

$$\eta = \left(\frac{U}{\nu x}\right)^{1/2} y, \quad f(\eta) = \frac{\psi}{(U\nu x)^{1/2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_0}, \tag{5}$$

where ψ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The introduction of the stream function automatically satisfies the continuity Eq. (1). Substituting (5) into Eqs. (2) and (3) gives

$$f''' + ff'' + 1 - f'^2 + \lambda\theta = 0, \tag{6}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'(S + \theta) = 0, \tag{7}$$

where primes denote differentiation with respect to η, $Pr = \nu/\alpha$ is the Prandtl number and $\lambda = \pm Gr_x/Re_x^2$ [with "±" sign has the same meaning as in Eq. (2)] is the buoyancy or mixed convection parameter. Further, $Gr_x = g\beta(T_w - T_0)x^3/\nu^2$ and $Re_x = Ux/\nu$ are the local Grashof number and the local Reynolds number, respectively. We notice that λ is independent of x, with $\lambda = +Gr_x/Re_x^2 > 0$ and $\lambda = -Gr_x/Re_x^2 < 0$ correspond to the assisting and opposing flows, respectively, while $\lambda = 0$ represents the pure forced convection flow. The boundary conditions (4) now become

$$f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1 - S, \\ f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \tag{8}$$

where $S = c/b$ is the constant stratification parameter. We notice that $S > 0$ implies a stably stratified environment, while $S = 0$ corresponds to an unstratified environment. It is worth mentioning that when $S = 0$, Eqs. (6)–(8) reduce to those derived by Ramachandran et al. [7] for the case of an arbitrary surface temperature with $n = 1$ in their paper.

3. Discussion and conclusions

The ordinary differential Eqs. (6)–(8) have been solved numerically by means of an implicit finite-difference scheme known as the Keller-box method, which is described in [8]. The values of the local Nusselt number $-\theta'(0)$ are presented in Table 1, which are very well comparable with previously reported results from the open literature. Moreover, dual solutions are found to exist for all values of Pr considered, which is a new result that was not reported by previous authors.

The variations of the skin friction coefficient $f''(0)$ with buoyancy parameter λ for $S = 0$ and $S = 0.5$ are shown in Fig. 1, for $Pr = 1$. This figure shows that it is possible to obtain dual solutions of the similarity Eqs. (6)–(8) also for assisting flow ($\lambda > 0$), apart from those for opposing flow ($\lambda < 0$), that have been reported by Ramachandran et al. [7], Devi et al. [9] and Lok et al. [10]. For $\lambda > 0$, there is a favorable pressure gradient due to the buoyancy forces, which results in the flow being accelerated and consequently there is a larger skin friction coefficient than in the non-buoyant case ($\lambda = 0$). For negative values of λ, there is a critical value λ_c, with two solution branches for $\lambda > \lambda_c$, a saddle-node bifurcation at $\lambda = \lambda_c$ and no solutions for $\lambda < \lambda_c$.

The boundary layer separates from the surface at $\lambda = \lambda_c$, thus we are unable to get the solution for $\lambda < \lambda_c$ by using the boundary layer approximations. To obtain the solutions beyond this value, the full Navier–Stokes equations

Table 1
Values of $-\theta'(0)$ for various values of Pr when $S = 0$ and $\lambda = 1$ (assisting flow)

Pr	Ramachandran et al. [7]	Devi et al. [9]	Lok et al. [10]	Present results	
				Upper branch	Lower branch
0.7	0.7641	0.7641	0.764087	0.7641	1.0226
1	–	–	–	0.8708	1.1691
7	1.7224	1.7223	1.722775	1.7224	2.2192
10	–	–	–	1.9446	2.4940
20	2.4576	2.4574	2.458836	2.4576	3.1646
40	3.1011	–	3.103703	3.1011	4.1080
50	–	–	–	3.3415	4.4976
60	3.5514	3.5517	3.555404	3.5514	4.8572
80	3.9095	–	3.914882	3.9095	5.5166
100	4.2116	4.2113	4.218462	4.2116	6.1230

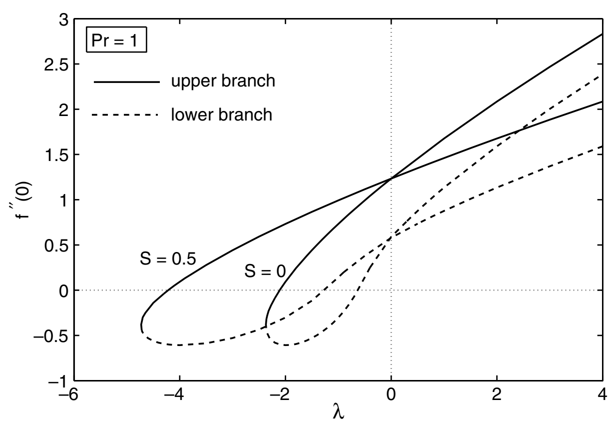

Fig. 1. Variation of $f''(0)$ with λ for $S = 0$ and $S = 0.5$ when $Pr = 1$.

have to be solved. Based on our computations, $\lambda_c = -2.364$ and $\lambda_c = -4.721$ for $S = 0$ and $S = 0.5$, respectively. The boundary layer separation occurs at $\lambda = \lambda_c$ where $f''(0) < 0$, a different result from the classical boundary layer theory where separation occurs when $f''(0) = 0$. This observation is in agreement with those reported by Ramachandran et al. [7], Devi et al. [9], Lok et al. [10], Schneider [11] and Schneider and Wasel [12]. Sears and Telionis [13] proposed that the name "separation" should not be given to vanishing wall-shear, $f''(0) = 0$. Further, Fig. 1 also shows that the thermal stratification significantly affects the surface shear stress, besides delays the boundary layer separation.

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